Explicit Expressions of the Reflection and Transmission for Two Coupled Identical Exponential Lines

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***Abstract*— The reflection and transmission problem of two coupled identical exponential lines is considered in this paper. The exact and explicit expressions for the co- and cross-line reflection and transmission coefficients are derived. The explicit expressions are validated by an independent numerical solution.**

***Index Terms*—Coupled transmission lines, distributed parameter circuits, exponential destributions, scattering matrices.**

I. Introduction

The analysis and various applications of exponential transmission lines (ETL's) have been investigated in numerous articles [1]–[4]. Coupled transmission lines have been of continuous interest for both microwave and power engineering (see [5], [6]). In microwave engineering, coupled lines have been used as directional couplers, phase shifters, filters, etc. Coupled nonuniform lines can offer a class of ultrabroad-band components due to the nature of nonuniformity, and thus improve the performance. In power engineering, the coupling between lines is, in general, nonuniform due to the complex geometries (see [7]).

Exact and explicit solutions for special cases are useful in investigating some possible applications, for the insight they provide, and as references in establishing benchmarks for general numerical algorithms. In this paper, one derives the exact and closed-form solution for the reflection and transmission problem of two coupled identical exponential lines through a transformation. The explicit expressions for the co- and cross-line reflection and transmission coefficients are then validated by a numerical solution.

II. Problem Formulation

Consider two coupled transmission lines of finite length l between $z = 0$ and $z = l$ (see Fig. 1 for the configuration). The equations for the voltage $v_k(z)$ and current $i_k(z)$, $k = 1, 2$, in the frequency-domain are as follows [for a harmonic time dependence $\exp(jwt)$] (see [8]):

$$\frac{d}{dz}\begin{bmatrix} v_1 \\ v_2 \\ i_1 \\ i_2 \end{bmatrix} = -j\omega \begin{bmatrix} 0 & \overline{\overline{L}}(z) \\ \overline{\overline{C}}(z) & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ i_1 \\ i_2 \end{bmatrix} \equiv D \begin{bmatrix} v_1 \\ v_2 \\ i_1 \\ i_2 \end{bmatrix} \tag{1}$$

where

$$\overline{\overline{L}}(z) = \begin{bmatrix} L_1(z) & L_m(z) \\ L_m(z) & L_2(z) \end{bmatrix}, \qquad \overline{\overline{C}}(z) = \begin{bmatrix} C_1(z) & -C_m(z) \\ -C_m(z) & C_2(z) \end{bmatrix} \tag{2}$$

and where $L_k(z)$, $C_k(z)$, $k = 1, 2$, are, respectively, the self-inductance and self-capacitance per-unit length at the position z of line k in the presence of the other line, and $L_m(z)$, $C_m(z)$ are the mutual inductance and mutual capacitance per-unit length at the position z, respectively. If the lines are lossless, all these parameters have real values. If the lines are lossy (i.e., have nonzero shunt conductance and/or resistance), then some of these parameters may have complex values. Equation (1) is a first-order linear system of ordinary differential equations (ODE's) with inhomogeneous coefficients. Generally speaking, a linear system of ODE's with *arbitrarily* inhomogeneous coefficients cannot be solved explicitly. Note that the explicit solution given by Nwoke in [9] for an arbitrarily nonuniform transmission line is not correct (as pointed out in [4] and [10], the theorem presented in [9] is incorrect when the system of differential equations has varying coefficients, which Nwoke was not aware of).

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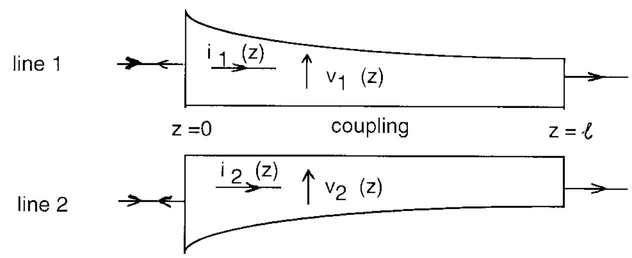

Fig. 1. Configuration for two coupled nonuniform transmission lines.

In this paper, the case of two coupled identical ETL's is considered, and thus [11]

$$\overline{\overline{L}}(z) = \begin{bmatrix} L_0 & L_{m0} \\ L_{m0} & L_0 \end{bmatrix} e^{2qz}$$
$$\overline{\overline{C}}(z) = \begin{bmatrix} C_0 & -C_{m0} \\ -C_{m0} & C_0 \end{bmatrix} e^{-2qz}, \qquad 0 < z < l \tag{3}$$

where L_0, C_0, L_{m0}, and C_{m0} are constants, and the constant q (which may be either positive or negative) defines the taper of the ETL's.

It is assumed that each line is connected to two uniform lossless *LC* lines at the two ends, and one has

$$\begin{aligned} L_k &= L_{00} \\ C_k &= C_{00} \\ L_m &= C_m \\ &= 0, \qquad k = 1, 2, \quad z < 0 \end{aligned} \tag{4}$$

$$\begin{aligned} L_k &= L_{l0} \\ C_k &= C_{l0} \\ L_m &= C_m \\ &= 0, \qquad k = 1, 2, \quad z > l. \end{aligned} \tag{5}$$

Note that all the parameters may have discontinuities at the endpoint $z = 0$ and $z = l$.

III. Internal Voltages and Currents

In this section, a suitable transformation of variables is used which transforms the system of ODE's for two coupled ETL's to a simple one with constant coefficients.

The following transformation (which comes from the diagonalization of the matrix D) are introduced:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}(z) = \frac{1}{2}\begin{bmatrix} 1 & 1 & Z_1 & Z_1 \\ 1 & -1 & Z_2 & -Z_2 \\ 1 & 1 & -Z_1 & -Z_1 \\ 1 & -1 & -Z_2 & Z_2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ i_1 \\ i_2 \end{bmatrix} \equiv P \begin{bmatrix} v_1 \\ v_2 \\ i_1 \\ i_2 \end{bmatrix} \tag{6}$$

where

$$Z_1 = \sqrt{\frac{L+L_m}{C-C_m}} = \sqrt{\frac{L_0+L_{m0}}{C_0-C_{m0}}}e^{2qz} \tag{7}$$

$$Z_2 = \sqrt{\frac{L-L_m}{C+C_m}} = \sqrt{\frac{L_0-L_{m0}}{C_0+C_{m0}}}e^{2qz}. \tag{8}$$

Differentiating (6) with respect to z and using (1), yields

$$\frac{d}{dz}\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix} = \left[PDP^{-1} + \left(\frac{d}{dz}P\right)P^{-1}\right]\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix} \tag{9}$$

where P^{-1} is the inverse of P, i.e.,

$$P^{-1} = \frac{1}{2}\begin{bmatrix}1 & 1 & 1 & 1\\ 1 & -1 & 1 & -1\\ \frac{1}{Z_1} & \frac{1}{Z_2} & \frac{-1}{Z_1} & \frac{-1}{Z_2}\\ \frac{1}{Z_1} & \frac{-1}{Z_2} & \frac{-1}{Z_1} & \frac{1}{Z_2}\end{bmatrix}. \tag{10}$$

After a matrix calculation, one obtains the following system of equations:

$$\begin{aligned}\frac{d}{dz}\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix} &= \begin{bmatrix}\frac{-j\omega}{c_1}+q & 0 & -q & 0\\ 0 & \frac{-j\omega}{c_2}+q & 0 & -q\\ -q & 0 & \frac{j\omega}{c_1}+q & 0\\ 0 & -q & 0 & \frac{j\omega}{c_2}+q\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix}\\ &\equiv A_0\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix}\end{aligned} \tag{11}$$

where the constants

$$c_1 = \frac{1}{\sqrt{(L+L_m)(C-C_m)}} = \frac{1}{\sqrt{(L_0+L_{m0})(C_0-C_{m0})}} \tag{12}$$

$$c_2 = \frac{1}{\sqrt{(L-L_m)(C+C_m)}} = \frac{1}{\sqrt{(L_0-L_{m0})(C_0+C_{m0})}}. \tag{13}$$

Note that the system (11) is a first-order linear system of ODE's with *constant* coefficients.

The system (11) of ODE's (with constant coefficients) can be solved in a conventional way [12]. The final result is given as follows:

$$\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix}(z) = e^{qz}A(z)\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix}(0^+), \qquad 0 < z < l \tag{14}$$

where

$$A(z) = \begin{bmatrix}\overline{\overline{A}}_1 & \overline{\overline{A}}_2\\ \overline{\overline{A}}_2 & \overline{\overline{A}}_3\end{bmatrix} \tag{15}$$

and as shown in (16)–(20) at the bottom of the page.

Therefore, the internal voltages and currents are given by the following equation:

$$\begin{bmatrix}v_1\\v_2\\i_1\\i_2\end{bmatrix}(z) = e^{qz}[P^{-1}(z)][A(z)][P(0^+)]\begin{bmatrix}v_1\\v_2\\i_1\\i_2\end{bmatrix}(0), \qquad 0 < z < l. \tag{21}$$

IV. Reflection and Transmission

It is assumed that the excitation voltage is from the left side of the ETL's, i.e., $z \le 0$. The (right-moving) incident voltage $v_k^{\rm inc}$, $k = 1, 2$, and the (left-moving) reflected voltage $v_k^{\rm refl}$, $k = 1, 2$, at $z = 0^-$ on the kth line are given by [13]

$$\begin{bmatrix}v_1^{\rm inc}\\v_2^{\rm inc}\\v_1^{\rm refl}\\v_2^{\rm refl}\end{bmatrix} = \frac{1}{2}\begin{bmatrix}I & \sqrt{\frac{L_{00}}{C_{00}}}I\\ I & -\sqrt{\frac{L_{00}}{C_{00}}}I\end{bmatrix}\begin{bmatrix}v_1\\v_2\\i_1\\i_2\end{bmatrix}(0) \equiv T_0\begin{bmatrix}v_1\\v_2\\i_1\\i_2\end{bmatrix}(0) \tag{22}$$

where I is the 2×2 unit matrix. Since there is no left-moving wave in the region $z > l$, one has

$$\begin{bmatrix}v_1^{tr}\\v_2^{tr}\\0\\0\end{bmatrix} = \frac{1}{2}\begin{bmatrix}I & \sqrt{\frac{L_{l0}}{C_{l0}}}I\\ I & -\sqrt{\frac{L_{l0}}{C_{l0}}}I\end{bmatrix}\begin{bmatrix}v_1\\v_2\\i_1\\i_2\end{bmatrix}(l) \equiv T_l\begin{bmatrix}v_1\\v_2\\i_1\\i_2\end{bmatrix}(l) \tag{23}$$

$$\overline{\overline{A}}_1 = \begin{bmatrix}\cosh(\lambda_1 z) - j\frac{\omega}{c_1\lambda_1}\sinh(\lambda_1 z) & 0\\ 0 & \cosh(\lambda_2 z) - j\frac{\omega}{c_2\lambda_2}\sinh(\lambda_2 z)\end{bmatrix} \tag{16}$$

$$\overline{\overline{A}}_2 = \begin{bmatrix}-\frac{q}{\lambda_1}\sinh(\lambda_1 z) & 0\\ 0 & -\frac{q}{\lambda_2}\sinh(\lambda_2 z)\end{bmatrix} \tag{17}$$

$$\overline{\overline{A}}_3 = \begin{bmatrix}\cosh(\lambda_1 z) + j\frac{\omega}{c_1\lambda_1}\sinh(\lambda_1 z) & 0\\ 0 & \cosh(\lambda_2 z) + j\frac{\omega}{c_2\lambda_2}\sinh(\lambda_2 z)\end{bmatrix} \tag{18}$$

$$\lambda_1 = \left(q^2 - \frac{\omega^2}{c_1^2}\right)^{1/2} \tag{19}$$

$$\lambda_2 = \left(q^2 - \frac{\omega^2}{c_2^2}\right)^{1/2}. \tag{20}$$

where v_k^{tr}, $k = 1, 2$, is the (right-moving) transmitted voltage at $z = l^+$ on the kth line.

From (21)–(23) it follows that

$$\begin{bmatrix} v_1^{tr} \\ v_2^{tr} \\ 0 \\ 0 \end{bmatrix} = e^{qz} T_l [P^{-1}(l^-)][A(l^-)][P(0^+)] T_0^{-1} \begin{bmatrix} v_1^{\rm inc} \\ v_2^{\rm inc} \\ v_1^{\rm refl} \\ v_2^{\rm refl} \end{bmatrix} \equiv e^{ql} \begin{bmatrix} \overline{\overline{a}} & \overline{\overline{b}} \\ \overline{\overline{m}} & \overline{\overline{n}} \end{bmatrix} \begin{bmatrix} v_1^{\rm inc} \\ v_2^{\rm inc} \\ v_1^{\rm refl} \\ v_2^{\rm refl} \end{bmatrix} \tag{24}$$

where T_0^{-1} is the inverse of T_0, and $\overline{\overline{a}}$, $\overline{\overline{b}}$, $\overline{\overline{m}}$, and $\overline{\overline{n}}$ are 2 × 2 matrices. After a matrix calculation, one obtains

$$\overline{\overline{a}} = \frac{1}{4}\left\{ \overline{\overline{e}}\,\overline{\overline{a}}_1 \overline{\overline{e}} + \sqrt{\frac{C_{00}}{L_{00}}} \overline{\overline{e}}\,\overline{\overline{a}}_2 \overline{\overline{Z}}_0 + 2\sqrt{\frac{L_{l0}}{C_{l0}}} \overline{\overline{Z}}_l^{-1} \overline{\overline{a}}_2 \overline{\overline{e}} + 2\sqrt{\left(\frac{L_{l0}}{L_{00}}\right)\left(\frac{C_{00}}{C_{l0}}\right)} \overline{\overline{Z}}_l^{-1} \overline{\overline{a}}_3 \overline{\overline{Z}}_0 \right\} \tag{25}$$

$$\overline{\overline{b}} = \frac{1}{4}\left\{ \overline{\overline{e}}\,\overline{\overline{a}}_1 \overline{\overline{e}} - \sqrt{\frac{C_{00}}{L_{00}}} \overline{\overline{e}}\,\overline{\overline{a}}_2 \overline{\overline{Z}}_0 + 2\sqrt{\frac{L_{l0}}{C_{l0}}} \overline{\overline{Z}}_l^{-1} \overline{\overline{a}}_2 \overline{\overline{e}} - 2\sqrt{\left(\frac{L_{l0}}{L_{00}}\right)\left(\frac{C_{00}}{C_{l0}}\right)} \overline{\overline{Z}}_l^{-1} \overline{\overline{a}}_3 \overline{\overline{Z}}_0 \right\} \tag{26}$$

$$\overline{\overline{m}} = \frac{1}{4}\left\{ \overline{\overline{e}}\,\overline{\overline{a}}_1 \overline{\overline{e}} + \sqrt{\frac{C_{00}}{L_{00}}} \overline{\overline{e}}\,\overline{\overline{a}}_2 \overline{\overline{Z}}_0 - 2\sqrt{\frac{L_{l0}}{C_{l0}}} \overline{\overline{Z}}_l^{-1} \overline{\overline{a}}_2 \overline{\overline{e}} - 2\sqrt{\left(\frac{L_{l0}}{L_{00}}\right)\left(\frac{C_{00}}{C_{l0}}\right)} \overline{\overline{Z}}_l^{-1} \overline{\overline{a}}_3 \overline{\overline{Z}}_0 \right\} \tag{27}$$

$$\overline{\overline{n}} = \frac{1}{4}\left\{ \overline{\overline{e}}\,\overline{\overline{a}}_1 \overline{\overline{e}} - \sqrt{\frac{C_{00}}{L_{00}}} \overline{\overline{e}}\,\overline{\overline{a}}_2 \overline{\overline{Z}}_0 - 2\sqrt{\frac{L_{l0}}{C_{l0}}} \overline{\overline{Z}}_l^{-1} \overline{\overline{a}}_2 \overline{\overline{e}} + 2\sqrt{\left(\frac{L_{l0}}{L_{00}}\right)\left(\frac{C_{00}}{C_{l0}}\right)} \overline{\overline{Z}}_l^{-1} \overline{\overline{a}}_3 \overline{\overline{Z}}_0 \right\} \tag{28}$$

where

$$\overline{\overline{e}} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, \quad \overline{\overline{Z}}_0 = \begin{bmatrix} Z_1 & Z_1 \\ Z_2 & -Z_2 \end{bmatrix}(0^+), \quad \overline{\overline{Z}}_l^{-1} = \frac{1}{2} e^{-2ql} \begin{bmatrix} \frac{1}{Z_1} & \frac{1}{Z_2} \\ \frac{1}{Z_1} & \frac{-1}{Z_2} \end{bmatrix}(0^+) \tag{29}$$

$$\overline{\overline{a}}_1 = \begin{bmatrix} \cosh(\lambda_1 l) - \frac{q}{\lambda_1}\sinh(\lambda_1 l) & 0 \\ 0 & \cosh(\lambda_2 l) - \frac{q}{\lambda_2}\sinh(\lambda_2 l) \end{bmatrix} \tag{30}$$

$$\overline{\overline{a}}_2 = \begin{bmatrix} -i\frac{\omega}{c_1\lambda_1}\sinh(\lambda_1 l) & 0 \\ 0 & -i\frac{\omega}{c_2\lambda_2}\sinh(\lambda_2 l) \end{bmatrix} \tag{31}$$

$$\overline{\overline{a}}_3 = \begin{bmatrix} \cosh(\lambda_1 l) + \frac{q}{\lambda_1}\sinh(\lambda_1 l) & 0 \\ 0 & \cosh(\lambda_2 l) + \frac{q}{\lambda_2}\sinh(\lambda_2 l) \end{bmatrix}. \tag{32}$$

The reflection and transmission coefficient matrices are defined as usual in the following:

$$\begin{bmatrix} v_1^{\rm refl} \\ v_2^{\rm refl} \end{bmatrix} = \overline{\overline{r}} \begin{bmatrix} v_1^{\rm inc} \\ v_2^{\rm inc} \end{bmatrix}, \qquad \begin{bmatrix} v_1^{tr} \\ v_2^{tr} \end{bmatrix} = \overline{\overline{t}} \begin{bmatrix} v_1^{\rm inc} \\ v_2^{\rm inc} \end{bmatrix}. \tag{33}$$

It thus follows from (24) that

$$\overline{\overline{r}} = -\overline{\overline{n}}^{-1}\overline{\overline{m}} \tag{34}$$

$$\overline{\overline{t}} = e^{ql}(\overline{\overline{a}} + \overline{\overline{b}}\,\overline{\overline{r}}) = e^{ql}(\overline{\overline{a}} - \overline{\overline{b}}\,\overline{\overline{n}}^{-1}\overline{\overline{m}}). \tag{35}$$

Equations (34) and (35) give the exact and explicit solution for the reflection and transmission problem of two coupled identical exponential lines [the 2 × 2 matrices $\overline{\overline{a}}$, $\overline{\overline{b}}$, $\overline{\overline{m}}$, and $\overline{\overline{n}}$ are given by (25)–(28)].

As expected from the symmetry of the two coupled identical lines, the reflection and transmission matrices [given by the expressions (34) and (35)] have the following form:

$$\overline{\overline{r}} = \begin{bmatrix} r_{\rm co} & r_{\rm cross} \\ r_{\rm cross} & r_{\rm co} \end{bmatrix} \tag{36}$$

$$\overline{\overline{t}} = \begin{bmatrix} t_{\rm co} & t_{\rm cross} \\ t_{\rm cross} & t_{\rm co} \end{bmatrix} \tag{37}$$

where $r_{\rm co}$, $r_{\rm cross}$ ($t_{\rm co}$, $t_{\rm cross}$) are co- and cross-line reflection (transmission) coefficients, respectively. The physical interpretation of these quantities is as follows: $r_{\rm co}$ ($r_{\rm cross}$) is the amplitude of the left-moving reflected voltage at $z = 0^-$ on a line due to a unit amplitude of right-moving incident voltage at $z = 0^-$ on the line (or the other line) with the right-moving incident voltage on the other line put equal to zero. $t_{\rm co}$ and $t_{\rm cross}$ can be interpreted similarly. These reflection and transmission coefficients are plotted in Fig. 2, as functions of the taper parameter q (scaled by $1/l$) when $C_0 = C_{l0} = C_{00}$, $C_{m0} = 0.5C_{00}$, $L_0 = 3L_{00}$, $L_{l0} = 0.8L_{00}$, $L_{m0} = 2L_{00}$, and the frequency $\omega = 0.1/l\sqrt{L_{00}C_{00}}$. As one can see from Fig. 2, when $|q|$ becomes large, $|r_{\rm co}|$ approaches to 1, and $|r_{\rm cross}|$, $|t_{\rm co}|$, and $|t_{\rm cross}|$ approach to 0. The marks in Fig. 2(a) and (b) are the corresponding co- and cross-line scattering coefficients obtained by solving (numerically) the ODE's for the reflection-coefficient matrix and the Green's functions [14]. These marks are almost on the curves in Fig. 2, which show that the explicit solution is consistent with the numerical results obtained by solving the ODE's for the reflection-coefficient matrix and the Green's functions.

A Special Case: In a very special case when $C_m/C = L_m/L$ and the terminations are matched (i.e., $L_{00} = L_0$, $C_{00} = C_0$, $L_{l0} = L_0 e^{2ql}$, and $C_{l0} = C_0 e^{-2ql}$), one has $c_2 = c_1$, $\lambda_2 = \lambda_1$, $\sqrt{Z_1 Z_2} = \sqrt{L_0/C_0}e^{2qz}$. Thus, (34) gives

$$r_{\rm co} = \frac{\frac{q}{\lambda_1}\sinh(\lambda_1 l)}{\cosh(\lambda_1 l) + i\frac{\omega}{2c_1\lambda_1}\left[\sqrt{\frac{Z_1(0^+)}{Z_2(0^+)}} + \sqrt{\frac{Z_2(0^+)}{Z_1(0^+)}}\right]\sinh(\lambda_1 l)}$$

$$r_{\rm cross} = \frac{i\frac{\omega}{2c_1\lambda_1}\left[\sqrt{\frac{Z_1(0^+)}{Z_2(0^+)}} - \sqrt{\frac{Z_2(0^+)}{Z_1(0^+)}}\right]\sinh(\lambda_1 l)}{\cosh(\lambda_1 l) + i\frac{\omega}{2c_1\lambda_1}\left[\sqrt{\frac{Z_1(0^+)}{Z_2(0^+)}} + \sqrt{\frac{Z_2(0^+)}{Z_1(0^+)}}\right]\sinh(\lambda_1 l)}.$$

As expected, when $C_m = L_m = 0$ [and thus $Z_1(0^+) = Z_2(0^+)$] the above-mentioned two equations show that $r_{\rm cross} = 0$ and $r_{\rm co}$ is identical to the reflection coefficient for a single ETL [15].

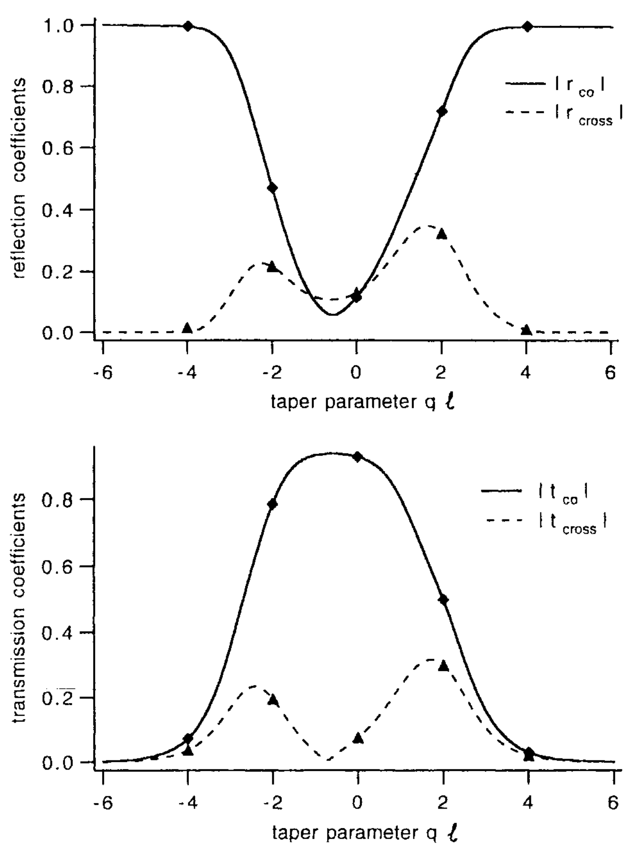

Fig. 2. The co- and cross-line reflection and transmission coefficients given by the explicit solutions (49) and (50) for two coupled identical exponential lines with the parameters $C_0 = C_{l0} = C_{00}$, $C_{m0} = 0.5C_{00}$, $L_0 = 3L_{00}$, $L_{l0} = 0.8L_{00}$, $L_{m0} = 2L_{00}$ and the frequency $\omega = 0.1/l\sqrt{L_{00}C_{00}}$. The marks in the figure are the corresponding co- and cross-line scattering coefficients obtained by solving numerically the ODE's for the reflection-coefficient matrix and the Green's functions.

V. Conclusion

The exact and explicit expressions for the co- and cross-line reflection and transmission coefficients for two coupled identical exponential lines have been derived. The explicit expressions have been validated by a numerical solution based on the wave-splitting technique.

References

[1] R. N. Ghose, "Exponential transmission lines as resonators and transformers," *IRE Trans. Microwave Theory Tech.*, vol. MTT-5, pp. 213–217, July 1957.
[2] C. P. Womack, "The use of exponential lines as microwave components," *IRE Trans. Microwave Theory Tech.*, vol. MTT-10, pp. 124–217, Mar. 1962.
[3] N. H. Younan, B. L. Cox, C. D. Taylor, and W. D. Prather, "An exponentially taped transmission line antenna," *IEEE Trans. Electromag. Compat.*, vol. 36, pp. 141–144, May 1994.
[4] S. He, "Closed-form solution for lossy exponential transmission line problem in frequency and time domains," *J. Electromag. Waves Appl.*, vol. 9, no. 4, pp. 521–540, 1995.
[5] G. Matthaei, L. Young, and E. M. T. Jones, *Microwave Filters, Impedance-Matching Networks, and Coupling Structures.* Norwood, MA: Artech House, 1980.
[6] J. R. Griffith and M. S. Nakhla, "Time-domain analysis of lossy coupled line system," *IEEE Trans. Microwave Theory Tech.*, vol. 38, pp. 1480–1487, Oct. 1990.
[7] C. E. Baum, "From the electromagnetic pulse to high-power electromagnetics," *Proc. IEEE*, vol. 80, pp. 789–817, June 1992.
[8] M. K. Krage and G. I. Haddad, "Characteristics of the coupled microstrip transmission lines-I: Coupled-mode formulation of inhomogeneous lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-18, pp. 217–222, Apr. 1970.
[9] C. Nwoke, "An exact solution for the nonuniform transmission line problem," *IEEE Trans. Microwave Theory Tech.*, vol. 38, pp. 944–946, 1990.
[10] S. Amari, "Comments on an exact solution for nonuniform transmission line problem," *IEEE Trans. Microwave Theory Tech.*, vol. 39, pp. 611–612, Mar. 1991.
[11] J. L. Hill and D. Mathews, "Transient analysis of systems with exponential transmission lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-25, pp. 777–783, Sept. 1977.
[12] J. K. Hale, *Ordinary Differential Equations.* New York: Wiley, 1969, pp. 93–100.
[13] S. He and S. Ström, "Time-domain wave-splitting approach to transmission along a nonuniform LCRG line," *J. Electromag. Waves Appl.*, vol. 6, no. 8, pp. 995–1014, 1992.
[14] S. He, "A time-harmonic Green's function technique and wave propagation in a stratified nonreciprocal chiral slab with multiple discontinuities," *J. Math. Phys.*, vol. 33, no. 12, pp. 4103–4110, 1992.
[15] R. P. Arnold, W. L. Bailey, and R. L. Vaitkus, "Normalized impedance graphs for exponential transmission line," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-22, pp. 964–965, Nov. 1974.

Operation of New Type Field Displacement Isolator in Ridged Waveguide

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***Abstract*—A new type ridged-waveguide field-displacement isolator is analyzed in this paper. Experimental results have been obtained for the isolation, insert loss, and voltage standing wave ratio (VSWR) in C- and X-band. The isolation and bandwidth are found to increase obviously.**

***Index Terms*—Isolator, ridged waveguide.**

I. Introduction

In 1960, Chen [1] proposed the experimental results of resonating isolator and field displacement isolator in a single ridged waveguide. He got useful results of the resonating isolator. As to the field displacement isolator, his experiments were failures. He found "the forward loss of the field displacement isolator in single ridged waveguide became nearly identical with the reverse loss, nonreciprocal effect was not distinct. Take out the resistance sheet, nonreciprocality couldn't be improved$\cdots$." Based on the study of [2]–[5], the authors think that Chen's failure is due to the asymmetry of the single ridge positioned in the waveguide, which caused asymmetrical field distribution, and to the spacing of the ridge from the ferrite, which was so far that the circular polarization field besides the ridge could not interfere with the magnetized ferrite. According to previous analysis, the authors propose a new type field displacement isolator in symmetrical ridged waveguide. The experimental work has shown

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